CST Transonic Optimization using Tranair++

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Design optimization using the Class-Shape-Transformation (CST) method suggested by Kulfan has been applied to 2-D airfoils and a 3-D wing. The CST method applies shape functions to an underlying transformation that mimics essential geometric features. As applied to airfoils, this transformation insures a round leading edge and sharp trailing edge as well as smooth geometry and the potential to reduce the number of necessary design variables for optimization. This study examines practical application of CST to transonic optimization using Tranair++. Among the issues examined is an evaluation of how many design variables are required and how CST should be applied to optimize a given geometry. Constraint implementation will also be explored to evaluate how well key aerodynamic features may be imposed in a CST framework. These issues will be explored first in 2D and then on a 3D wing with CST modeling in both chord-wise and span-wise directions.

I. Introduction

Aerodynamic design optimization requires intelligent geometric modeling to decrease the number of design variables and subsequent computational cost. Traditional geometric modeling has typically required a large number of design variables to model key aerodynamic features such as the round leading edge of an airfoil. The Class-Shape-Transformation (CST) method invented by Kulfan (Ref 1 & 2) promises to reduce the number of design variables necessary to produce robust optimized designs. It also promises several other benefits including insured smoothness, intuitive terms and robustness. This study examines practical application of CST modeling towards transonic optimization. We hope to determine the strengths and weaknesses and evaluate the relative merit of CST modeling for transonic optimization.

The objectives of this study are:

1) Demonstration of CST optimization on a 2D transonic airfoil.
2) Evaluation of the optimum number of design variables in a CST framework.
3) Constraint implementation.
4) Robustness of CST implementation.
5) Multi-point compared with single-point CST optimization.
6) 3D optimization of a wing alone geometry.

II. Method Description

A. CST Geometry Framework

The Class-Shape-Transformation (CST) method was invented by Kulfan (Ref 1 & 2) as a means to model generalized geometry. The method employs an underlying transformation to mimic essential geometric features and shape functions that build off that transformation to model specific geometry. The transformation is based on ‘Class’ functions formulated to define fundamental groups of geometric features such as an airfoils with a round leading edge and a sharp trailing edge and ‘Shape’ functions that specify unique geometric details within the fundamental ‘Class’ function. The general CST modeling for an airfoil is composed as follows:

$$\zeta(\psi) = \sqrt{\psi} \bullet (1-\psi) \bullet S_\psi + \psi \bullet \Delta\zeta_T$$

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Where:

$$S_\psi = \Sigma A_i K_i \psi^i (1-\psi)^{(n-i)}$$
$$K_i = n!/(i!(n-i)!)$$
$$\psi = x/c ; \zeta = z/c ; \zeta_T = \Delta \zeta_{TE}/c$$

Bernstein polynomials were selected for the shape function because they essentially define a composite set of smooth airfoils. The “partition of unity” characteristic of Bernstein polynomials together with the analytic nature of the shape function insures that this process captures the entire design space of smooth airfoils. It is recognized that alternate shape functions may be preferable and the selection of Bernstein polynomials was chosen as a starting point.

B. Computational and Optimization Method

Tranair++ is a full potential solver with coupled boundary layer and Cartesian grid adaptation. It was developed by Forrester Johnson and others at Boeing and is documented in References 3-4. Additional information is available on the Calmar Research Corporation web site (Reference 5) where is is licensed for commercial resale. The design optimization features in Tranair++ allow constrained multi-point design and have been used on several Boeing aircraft. Tranair++ allows user-defined movement, constraint and objective functions including inequality constraints. Figure 1 shows Mach contours on an RAE2822 solution including the grid adaptation.

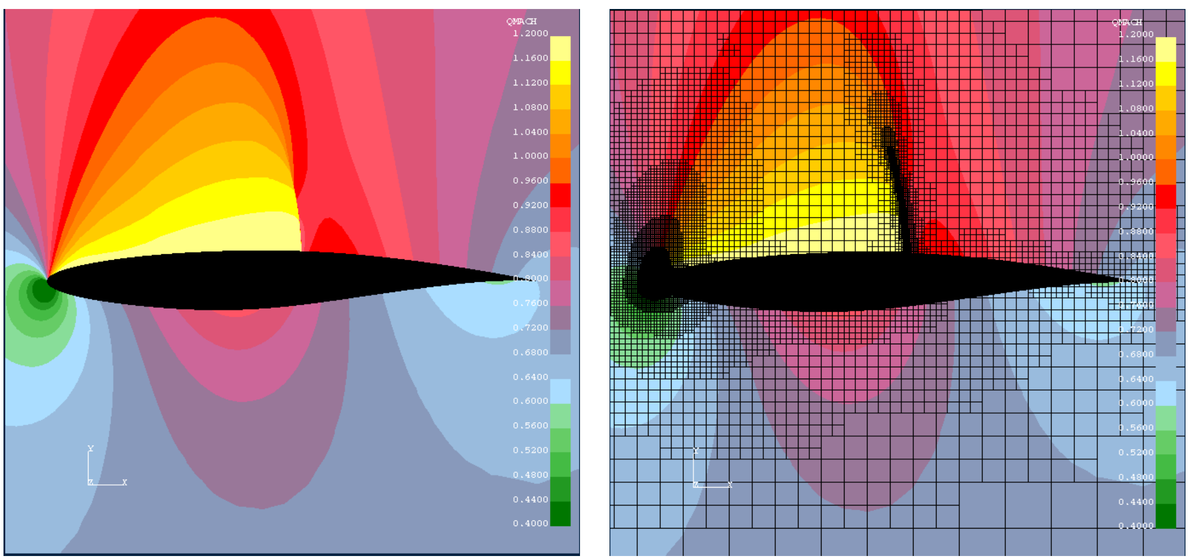

Figure 1. Tranair Mach Contours around RAE 2822 Airfoil with and without Grid

III. 2D Optimization

A. Design Modes Using CST

Tranair++ utilizes user defined design modes to move the geometry and optimize the design. This allows the aircraft designer complete control over design movement while reducing the number of design variables. CST optimization may be implemented either as the perturbation to an existing geometry or directly in 2D or 3D space. Direct CST optimization utilizes the CST shape functions as design variables so that every resulting airfoil is a CST airfoil. The perturbation method optimizes the sum of an existing geometry and a CST shape function optimization so that each delta will be defined by a CST airfoil. Since every airfoil may be mathematically described as a CST airfoil, these two methods are theoretically equivalent, although the number of design variables needed may not be practical.

A hybrid method combining the two is also possible. The addition of another design variable (λ) to the perturbation formula enables the baseline geometry to be scaled and potentially zeroed out if the new design variable is set to zero as given in the formula below. This method will likely provide additional robustness since it can reduce or eliminate adverse characteristics in the baseline geometry while providing the benefit of localizing the optimization.

Perturbation Movement: $\zeta_{NEW} = \zeta + \Delta\zeta_{CST}$

Direct Movement: $\zeta_{NEW} = \zeta_{CST}$

Hybrid Movement: $\zeta_{NEW} = \lambda\zeta + \Delta\zeta_{CST}$

The strengths and weaknesses of direct vs perturbation formulation are outlined in Table 1. The benefit of using a direct formulation is that resulting geometry will necessarily be smooth and that strategic terms may be imposed directly in movement or constraint routines as shape function coefficients. One disadvantage is that source geometry must be pre-fit to determine CST coefficients that are then sent to the optimization routine as starting points. The advantage of using perturbation formulation is that the source airfoil may be input directly with generalized perturbations and that these perturbations may be used for similar airfoils and topologies. The hybrid formulation has the advantages of both direct and perturbation formulations and the disadvantage of an additional extra design variable.

Item	**Direct Formulation**	**Perturbation Formulation**
LE radius and TE angle	Directly formulated from design variables	Must extract base geometry values to specify LE radius and TE angles
Smoothness	Always Smooth	Base geometry must be smooth
Thickness	Constraint is a set of linear equations	Constraint is a set of linear equations
Initial Geometry	CST coefficients are extracted from source airfoil prior to optimization.	Geometry is input directly as discrete points.

Table1. Strengths and Weakness of Direct vs Perturbation Method

Initial comparisons shown in Figure 2 reveal minor differences between perturbation and direct CST implementations. Both optimizations reduce the profile drag and virtually eliminate the wave drag. The virtual elimination of wave drag is expected for the single-point-optimization used for initial assessments. The optimization was performed on the RAE 2822 airfoil with constrained Mach, lift and thickness distribution. Since both methods are theoretically equivalent it isn't surprising that they are nearly identical at first glance. It's worth noting that the camber optimization drag benefit is within ½ count of D.Young's Tranair optimization results on the same airfoil as reported in Reference 5.

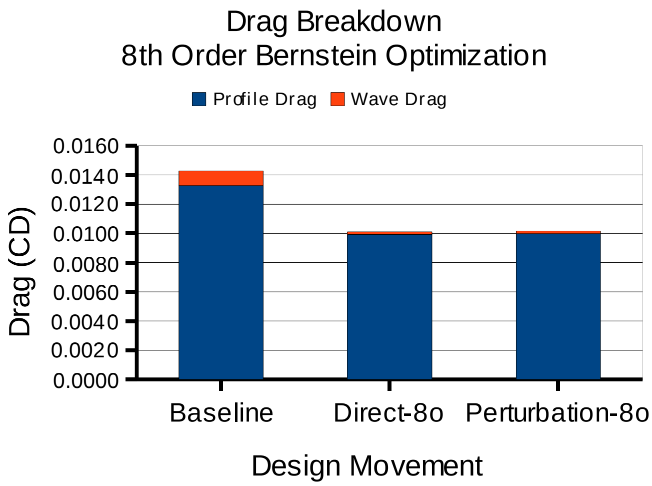

Figure 2. Comparison of CST Optimization Methods

B. Bernstein Polynomial Order

A sensitivity study was conducted to determine the effect of the Bernstein Polynomial Order on the optimization results. This study was conducted on the RAE 2822 airfoil at Mach 0.725 and Reynolds number 6.5M with both perturbation and direct CST movement. Lift was constrained to 0.73 and thickness distribution was constrained to match the RAE 2822. Results in Figure 3 show that minimum drag is generally achieved with 6 Bernstein polynomials and is constant within about 1 drag count for higher order Bernstein Polynomials. It is encouraging that the optimization asymptotes quickly and is relatively independent of the CST implementation.

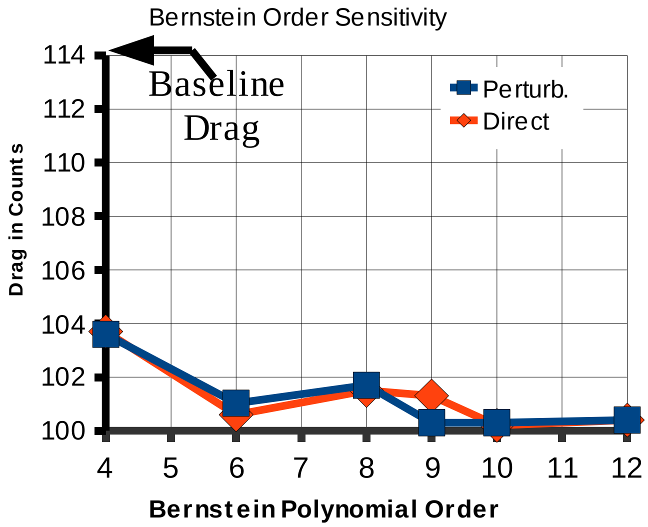

Figure 3. Bernstein Order Sensitivity

Baseline and optimized geometries are compared in Figure 4. Constraints limited optimization to camber modifications as shown in the figure. The resulting airfoil is shown to have more aft loading with a sharper leading edge peak and a mild double shock. These characteristics are the result of the single-point optimization used in the initial studies. Figure 5 shows a CST multi-point optimization that removes the double shock and leading edge peak observed in the single-point optimization. As is typical for multi-point optimization, the multi-point optimization retains 96% of the single-point optimization drag reduction.

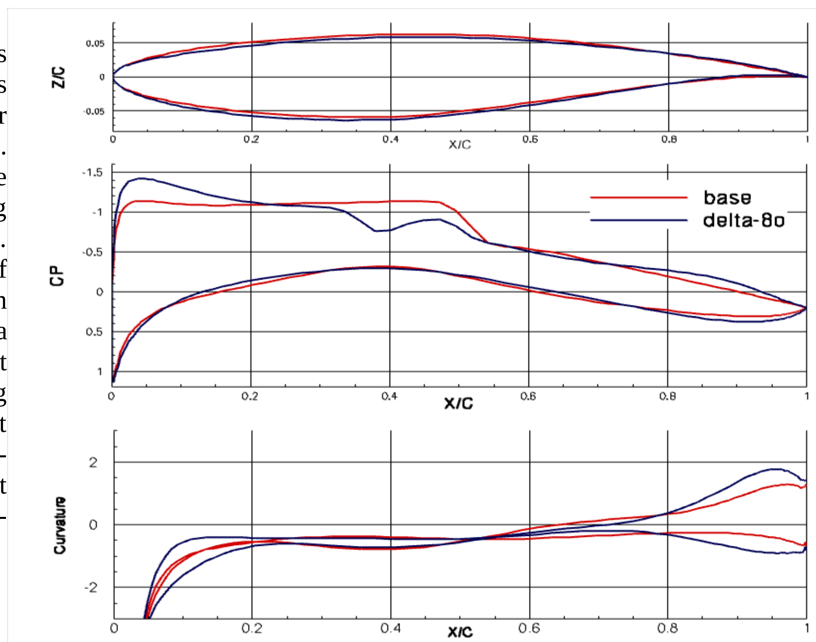

Figure 4. Single-Point Optimization Using CST

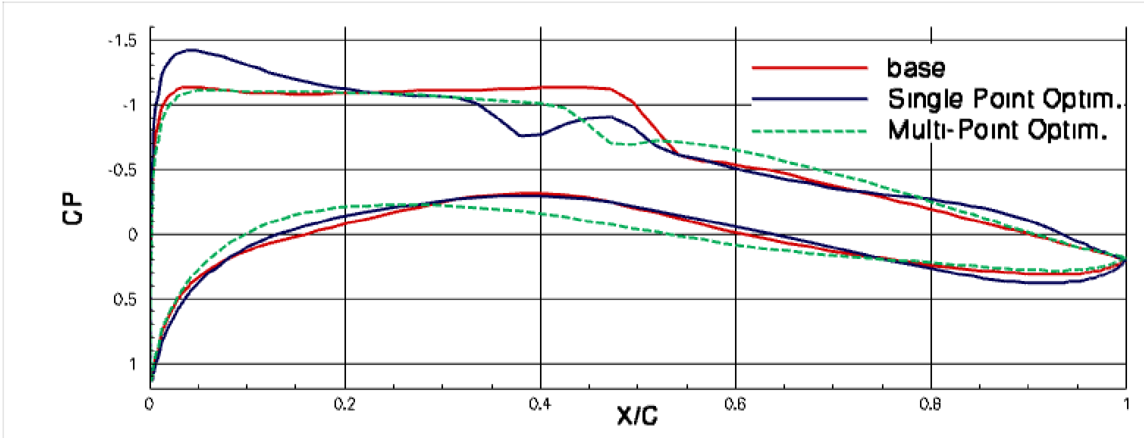

Figure 5. Multi-Point Optimization Pressure Architecture

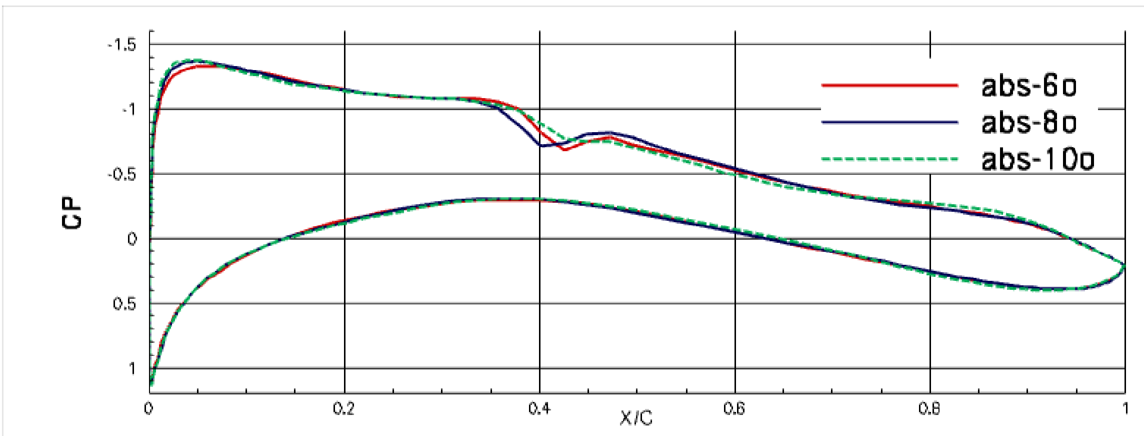

Figure 6. Effect of Bernstein Order on Single-Point Optimization Results

Figure 6 shows pressure architectures for 3 orders of Bernstein polynomial resulting from single-point optimization using direct CST implementation. The figure shows a slight reduction in the double shock and slight increase in aft loading as the Bernstein polynomial order is increased from 6 to 10. This illustrates the well-behaved nature of the CST formulation.

C. Geometry Constraints with CST

Constraint implementation in a CST framework offers several opportunities to impose constraints directly on the design variables. Several constraints may be described analytically with the potential advantage of simplifying optimization. Three constraints will be examined: leading edge radius, trailing edge angle, and spar thickness. The first two were used in our routines while the last was easier to implement with Tranair++'s constraints routine which allows inequality constraints on any variable that can be described.

1) Leading edge curvature (from Kulfan):

$$S_{\psi=0} = \sqrt{\frac{2 \bullet R_{LE}}{C}}$$

2) Trailing edge boat-tail angle (β) (from Kulfan):

$$S_{\psi=1} = \tan(\beta)$$

3) Spar thickness: Spar thickness constraints limit amplitudes of the system of Bernstein polynomial coefficients. For most applications where the Bernstein polynomial order is 6 or higher and the spar thickness is constrained at 2 chordwise locations, spar thickness constraints limit the system of equations and do not specify a single coefficient. The thickness constraint is given by the formula below:

$$\Delta\zeta = \sqrt{\psi}(1-\psi)\Sigma S_i (A_i^U - A_i^L)$$

$$S_i = (n!/(i!(n-i)!))\sqrt{\psi}(1-\psi)$$

A simplified means of achieving a similar result is to scale the design variables by the ratio of target/current thickness times the shape factor as given by the formula below:

$$\Delta\zeta = \Delta\zeta_o (thick_{target}/thick_{current})(S_i)$$

Where S_i is the shape function at the spar location being constrained.

Tranair++ features automatic and user defined constraint functions, and it turns out it is much easier to implement thickness and curvature constraints using the Tranair++ functionality than defining special functions derived from CST coefficients. This Tranair++ functionality is utilized for applying curvature and thickness constraints for the remainder of this study.

D. Robustness

Results shown in Figures 2-3 illustrate that optimized performance is generally insensitive to the Bernstein polynomial order or the implementation method, and all cases analyzed converged without great difficulty. In the course of the investigation, several issues were identified relating to the robustness of CST optimization.

1) The leading edge must be extracted precisely. Many CAD surfaces, such as the DLR-F6 wing, identify a leading edge that isn't precise enough for CST optimization. The wing should be discretized to place a point precisely on the leading edge for each airfoil across the span or risk the geometry crossing over itself for moderately blunt airfoils.
2) High order Bernstein polynomials are limited by numerical limitations. We observed round-off errors leading to failures for Bernstein polynomials greater than 14 orders magnitude. This isn't surprising considering the factorial computations needed to generate the Bernstein basis functions. Results indicate this is an insignificant limitation since lower order Bernstein polynomials appear to work just fine.
3) Higher order Bernstein polynomials converged with more difficulty than lower order polynomial optimizations. Results indicate convergence issues were related to the excitement of the single Bernstein polynomial near the base of the shock. The optimizer allowed a small bump in this region to pull the shock aft a little, and the higher order Bernstein polynomials accentuated the effect. Multi-point optimization would likely reduce or eliminate this problem.
4) Direct movement implementation had more convergence issues than perturbation movement. This was likely caused by the step size of the design variables allowed by the optimizer. Initial computations made with the absolute movement implementation allowed for larger design variable step sizes relative to the perturbation implementation due to user input. Reduction of the allowable design variable step size allowed the direct movement implementation to converge for all cases analyzed. More discipline is required to set the design variable range with the direct method relative to the perturbation method to maintain reasonable design variable step sizes.

IV. 3D Optimization

A. Geometry

Our initial assessment of 3D optimization utilized the wing from the DLR-F6 configuration analyzed in the Drag Prediction Workshops and detailed in Reference 6. We selected this wing since it is an industry standard and contained a planform break.

B. Optimization Method

Bernstein polynomials were used in both chord and spanwise directions. CST movement was proscribed as perturbations to the existing geometry in both chord and spanwise directions. Geometry movement was restricted normal to the chord plane. Figures 7-8 show results for 6 Bernstein polynomials in the chordwise direction and 5 in the spanwise direction. Constraints were imposed on the lift coefficient, spar thickness and boundary layer health.

C. Optimization Results

The optimization of the wing resulted in a drag reduction of 20 counts as shown in Figure 7. It's interesting to note that the optimizer strengthened the weak shock to gain an induced drag benefit. In this case the trade was advantageous to increase the wave drag slightly to improve the induced drag. Figure 8a and 8b show pressure contours for this case. The smooth isobars in the optimized case indicate the result should be relatively independent of the order of Bernstein polynomials used to model spanwise variations. This is a single-point optimization result.

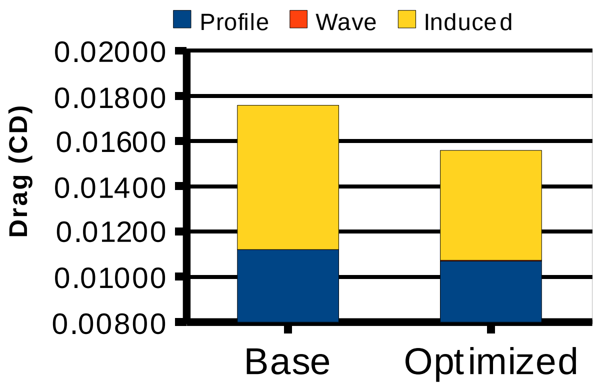

Figure 7. Breakdown of drag for 3D Optimization

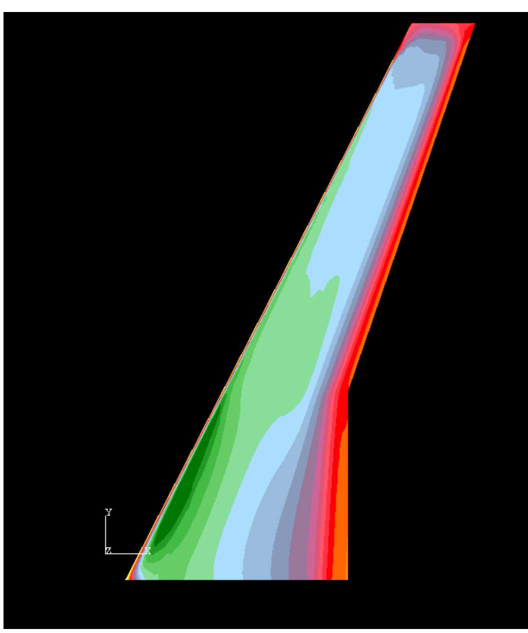

Figure 8a. Baseline DLR-F6 Wing

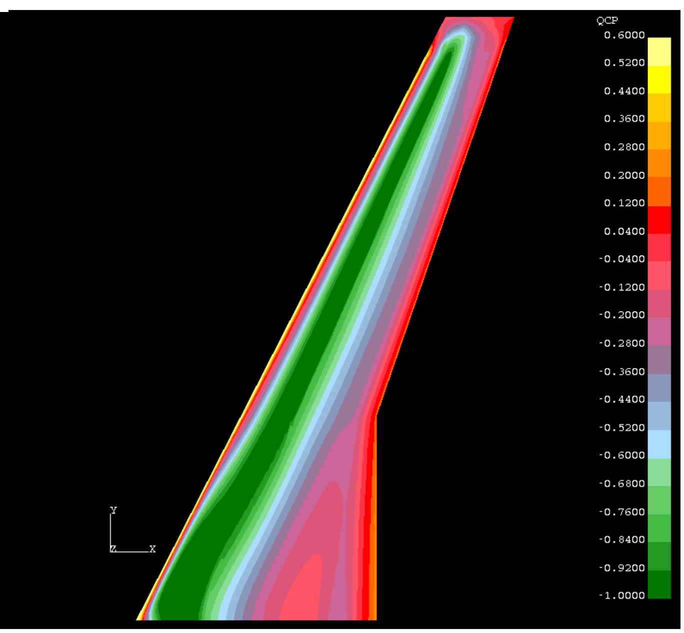

Figure 8b. Optimized DLR-F6 Wing

V. Summary and Conclusions

The CST method has been applied to 2D and 3D transonic optimization problems with success. Both direct and perturbation implementations resulted in consistent results and were generally insensitive to the Bernstein polynomial order as long as it was 6 or higher. 2D optimization results were consistent with conventional optimization used by Young in Reference 4. Several robustness issues were identified and resolved. In each case studied, the CST method performed as anticipated. Based on the results of this study, we recommend further investigation into CST optimization including the use of non-Bernstein basis functions targeted to key airfoil characteristics.

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